Closing Tuesday: 6.3, 6.4
Closing Thursday: 6.5
Final exam is Saturday, December 10
5:00pm to 7:50pm in Kane Hall 130.
I will email out a seating chart.

Entry Task:
Which account is best?
A: 4%, compounded semi-annually
B: 3.97%, compounded monthly
C: 3.955%, compounded continuously

The fast answer is to compute the annual percentage yield (APY) for each. Let me explain what APY is by doing the following:

Write out each formula, then plug in 1 year and simplify:

A:

B:

C:

6.3 and 6.4 Annuities

An **annuity** is an interest bearing account with regular deposits or withdrawals.

Two types of Annuities:

Ordinary Annuities = payments made at the END of each compounding period.

Annuities Due = payments made at the BEGINNING of each compounding period.

Two types of questions:

Future Value Questions = start with zero dollars in the account, make regular deposits, find the future value.

Examples: **Regular payments** into a retirement account, or an account to pay for college, in these **the account balance is growing**.

Present Value Questions = start with a lot of money (call this *P*) in the account, make regular withdrawals, then end with zero in the account.

Examples: Withdrawing from your retirement account, or paying down the balance of a loan, in these **account balance is shrinking**.

R = amount of each regular payment

r = decimal interest rate

m = num. of compoundings in a year

Compute:

$i = \frac{r}{m}$ = rate at each compounding

$n = mt$ = total payments

	Ordinary (Payments at END of each period)	**Due** (Payments at BEGINNING of each period)
FV (Balance Growing)	$F = R\frac{(1+i)^n - 1}{i}$	$F = R\frac{(1+i)^n - 1}{i}(1+i)$
PV (Balance Shrinking)	$P = R\frac{1-(1+i)^{-n}}{i}$	$P = R\frac{1-(1+i)^{-n}}{i}(1+i)$

Where do these formulas come from?
(You don't need to write this down).
First, you need to know the geometric sum.
By expanding you can see:

$$(1+x)(x-1) = x^2 - 1$$
$$(1+x+x^2)(x-1) = x^3 - 1$$
$$(1+x+x^2+x^3)(x-1) = x^4 - 1$$

and so on …

In each case, dividing by x – 1 gives

$$1+x = \frac{x^2-1}{x-1}$$
$$1+x+x^2 = \frac{x^3-1}{x-1}$$
$$1+x+x^2+x^3 = \frac{x^4-1}{x-1}$$

Thus, in general,

$$1+x+x^2+\cdots+x^{n-1} = \frac{x^n-1}{x-1}$$

For example:

$$1+(1.02)+\cdots+(1.02)^7 = \frac{(1.02)^8-1}{1.02-1}$$

Second, consider an annuity with regular payments at the end of each quarter for 2 years and 8%, compounded quarterly.

t = 2 years
r = 0.08, m = 4,
i = r/m = 0.02 (rate used each quarter)
n = mt = 8 payments

Map it out:

FROM THE LECTURE PACK:

1. At the end of each month, you place $100 into an account bearing 6% interest, compounded monthly. What is the balance of the account 5 years after you start?

Ordinary or Due? , FV or PV?

r = , m = , t =

i = , n = ,

R = , FV/PV =

2. A company establishes a sinking fund to pay a debt of $100,000 due in 4 years. At the beginning of each six-month period, they deposit $R in an account paying 9%, compounded semi-annually. How big must the payments be to pay the debt on time?

Ordinary or Due? , FV or PV?

r = , m = , t =

i = , n = ,

R = , FV/PV =

3. Your retirement account earns 7%, compounded quarterly. How much must the account contain when you retire if you want to withdraw $6000 at the end of each quarter for 30 years?

Ordinary or Due? , FV or PV?

r = , m = , t =

i = , n = ,

R = , FV/PV =

4. You inherit $200,000 and invest it at 3%, compounded monthly. If you withdraw $1000 at the beginning of every month, how long will the money last?

Ordinary or Due? , FV or PV?

r = , m = , t =

i = , n = ,

R = , FV/PV =